RESEARCH ARTICLE

CORRECTIVE LATERAL TRANSSHIPMENT APPLICATION IN A CENTRALIZED INVENTORY SYSTEM WITH RANDOM DEMAND: CASE STUDY

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Abstract : Our research work aims to study the lateral transshipment problem as a mode of cooperation between different retailers who are located at the same level.
In the first work, we apply a simulation-optimization approach based on a metamodel to search for the different measures of the initial level of replenishment.
Then, by applying a series of simulation experiments by the ARENA software to select the best lateral transshipment policy. This aims to maximize the expected average global gain and minimize the average global Disservice rate.
For this, several Transshipment policies will be tested, for example; non-pooling, full pooling and partial pooling policies depending on the selection of physical inventory thresholds.

Keyword: Transshipment policies,Simulation-approach, Complete-Pooling, Vendor-Managed Inventory, Supply Chain Management, Partial-Pooling Threshold.

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Introduction

In a supply chain, the distribution network is the main component through which the flow of products to customers is carried out. In the classic case, the network has three levels: "central warehouse, distribution centers and retailers", the latter being in direct contact with end customers. The problem of managing flows through such a network is obviously complex. First, it depends on the configuration of the network considered. Then, it must integrate the consideration of a multitude of parameters: number of products, demand law, desired service rate and the stock management policy at retailers and distribution centers, supply times, various costs (order, possession, transport and shortage). But, to be more competitive in a complex market, the supply chain can improve its performance by applying coordination between its different retailers, called "transshipment", which organizes stock transfers between sites of the same level, either according to a preventive policy to reduce the risks of shortages in the face of expected customer demands, or according to an emergency policy to resolve the problem of actual shortages. The importance of transshipment continues to increase, especially following the existence of a strategy called Vender Managed Inventory (VMI). According to this strategy, it is the supplier who manages the product stocks at the distributor and thus the procurement decision will be taken by the supplier and no longer by the customer.
The relationship between suppliers and customers has changed significantly: high quality requirements, product diversification, competitive costs, etc. These constraints have forced companies to seek new ways to improve their performance and better meet customer needs. Companies must question their organizational structures while ensuring a partnership between the different players. To do this, logistics helps improve the flow of flows between all these links from the supplier's supplier to the customer's customer. Logistics' mission is to ensure a dialogue between internal and external players in order to ensure good circulation of physical flows, information flows and financial flows.
Good supply chain management improves the management of physical and digital flows for the company and its customers. A guarantee of quality, competitiveness and compliance, it guarantees the best service at the lowest cost.
The supply chain is a series of internal processes, from placing the order to final delivery to the customer. It is all the steps necessary to transport an ordered product to the consumer. A complete supply chain extends well beyond the company's internal processes. Indeed, it must manage its relationships with its partners (customers, inventory managers, distributors, suppliers, consumers, etc.) On a daily basis, supply chain management is one

of the top concerns of business leaders. It involves, step by step, estimating the right added value to bring to the product based on the customer's expectations and your constraints. By being able to offer the best products at the lowest cost, the company uses its supply chain as a real competitive asset. This type of management gives companies better control over inventory costs, the supply of goods and transport costs. Let's go back over the fundamental principles of logistics management, and our advice for improving the functioning of this type of management within your company.

Dehghani and Abbasi (2018) studied a shelf-life-based lateral transshipment policy for the case of perishable products. They transported blood units between hospitals. They developed partial differential equations to derive and solve a joint distribution problem that allowed them to determine the optimal inventory level at each location with transshipment based on inventory age. They also showed that their approach could provide additional cost savings to a similarly structured distribution channel.

A large number of works focus in their research on the type of lateral transshipment, ie. Emergency transhipment or preventive transhipment.

Herer et *al.,* (2002) analyzed the emergency lateral transshipment strategy between two-retailers and found this last can simultaneously improve lightness and agility.

The research work of Archibald et al. (2009) aims to study a storage system, while the decision to place an emergency order from the central warehouse or to use lateral transfer depends on the costs, the time remaining in the warehouse and the distance between retailers. The assumption of instantaneous replenishment time of the central warehouse considerably complicates the mathematical analysis of the network due to the interrelationships between demand, quantities to be transferred and stock in transit. In particular, if the optimal transfer strategy must take into account both stocks on hand and those on order, this implies that the state space must be increased.

Li et *al*., (2013) aim to study the effect of preventive lateral transhipment on the quantities ordered in a two-echelons inventory system.

Paterson et *al.,*(2012) and Noham and Tzur (2014) respectively developed a quasi-myopic approach and a simple heuristic algorithm.

Due to the complexity of the decision space, the exact model will be solved by minimizing the number of retailers to only two. Liao et al. (2014) studied a relationship between lateral transfer with transshipment and emergency orders.

The transfer between retailers is done in a bidirectional transshipment manner to coordinate the transshipped quantities.

Olsson (2015) studied a lateral transshipment policy for a two-retailer inventory system with a positive transshipment lead time.

Lee and Park (2016) studied an inventory model with two retailers and a single supplier with uncertain capabilities. By applying lateral transshipment, they found that a transshipment price could help coordinate the supply chain. Feng. et al., (2017) discussed a dynamic preventive type lateral transshipment problem in a centralized inventory system based on Markov decision making.

We distinguish two transshipment approaches. The first is that of emergency transshipment (reactive transshipment); it corresponds to the Transshipment carried out following an actual stock shortage at a retailer resulting from the arrival of a demand. In the literature, several research studies aim to study this approach.

Van et al. (2009) studied the problem of a two-echelon inventory system. They applied the Markov process to solve it by applying preventive transshipment at a well-determined date. Paterson et al., (2010) analyzed a multi-warehouse inventory system that follows the stock policy (S-1, S) combined with the proactive transshipment policy. They assumed that the transshipment cost is fixed, and they aimed to fix the optimal time of stock redistribution to minimize the shortage, this leads to a minimization of the total cost.

Reyes et al., (2013) studied the same problem as (Paterson et al., 2012) focusing their research work on the impact of emergency transshipment on inventory management in this system in the event of an actual stockout, and they concluded that reactive transshipment can reduce costs and improve the service rate by minimizing the quantity of lost customer orders. The second approach is preventive transshipment (also called proactive), which corresponds to a redistribution of stocks at the beginning or end of each supply cycle but before the customer demand is realized. There is a vast literature that focuses on this type of transshipment approach. Research on preventive transshipment is dominated by periodic review, because at the beginning and end of each period it is necessary to make a periodic check on the quantities stored to assign a redistribution of these quantities. In this regard, Agrawal et al (2004) considered a two-echelon inventory system in which they aimed to rebalance the stock quantity at a predetermined time before the demand is realized and they presented a dynamic programming formulation to determine the best decisions.

To significantly improve a purely reactive transshipment policy, it would be possible to combine it with another proactive policy; This will be called "hybrid transshipment policy" (see, Archibald et al. (2014)). In this research area, there is a vast literature that focuses on hybrid transshipment policy (reactive and proactive), for example, (Paterson et al, (2011)) who studied the same problem as (Paterson et al. (2010)) but combined both types of transshipment (corrective and preventive), to improve customer service rate and Glazebrook et al. (2015) who studied the same problem as Reyes et al. (2013), with a proposal to use a hybrid transshipment policy (reactive and preventive) exploiting economies of scale by reducing total cost through improvement in transportation cost and inventory cost.

The research work of Nakandala et al. (2017) focuses on studying a corrective lateral transshipment model for perishable items in a supply chain.

The research of Silbermayr et al. (2017) will focus on studying the lateral transshipment problem with environmental sustainability. This research aims to apply a more comprehensive transition decision method to help the practitioner make cost-effective decisions. Feng et al. (2018) analyzed this advanced research to study emergency lateral transshipment cooperated with preventive transshipment in a comparable, partially delayed setting. Dehghani and Abbasi (2018) propose an emergency transshipment policy for perishable goods in supply chains. They developed a heuristic solution to calculate performance measures.

Meissner and Senicheva, (2018) studied a multi-site, multi-period storage system with proactive (preventive) transshipment and approximate dynamic programming used to determine an optimal order policy and transshipment policy.

Timajchi et al. (2019) analyzed pharmaceutical product spoilage and proposed a lateral shift option to meet demand while simultaneously minimizing costs and accidental losses. Yi et al., (2020) studied optimal lateral shift and replenishment decisions in a decentralized setting. We build a multi-stage stochastic model that captures demand uncertainty and customer switching behavior. We demonstrate that, as in the centralized case, the optimal transshipment decision follows a dual-threshold structure.The optimal replenishment quantities are determined under two pricing mechanismsindividual mechanism (IP) and negotiated mechanism (NP).

1. Materials and methods

In our work, we seek to improve the overall profitability of the stock system composed of two retailers by minimizing the average Desservicerate of the two warehouses. This results in the improvement of the average overall gain in the entire stock system, which can be done by applying the cooperation between these retailers of the same echelon, this is called the "Lateral Transshipment", either by applying the transshipment policy: "Full Pooling" or "Partial Pooling". Each time we modify the threshold of the transshipment policy "Partial Pooling", the periodicity T and the unit cost of the transshipment.

We consider a distribution system consisting of two retailers no-identical *(i*=1, 2) owned or operated by the same entity and one manufacturer that sells to these retailers in a single period. Following the newsvendor scenario, the central depot owner needs to decide, for retailer *n*, a no negative order quantity Q_i, before observing demand D_i, with *i*= 1,2. Let $Q = \sum_{n=1}^{2} Q_n$ and $D = \sum_{n=1}^{2} D_n$ We assume that the distribution center shares orders for a single product within a fixed period (control period), and this regardless of the quantities of supply.To solve this type of problem, we can apply the "Without-Transshipment" policy, that is to say, when the retailer falls into an out-of-stock position, he demands the quantity of central deposit missing to satisfy customer demand, or, by applying the "With-Transshipment" policy, by adopting a relationship between the retailers who are in the same line to minimize the stock shortage and meet a random demand.In this paper we will try to find the most appealing controversy that aims to maximize the expected Average Global Profit and minimize as much as possible the Average Global Desservice Rate.

Results and discussion

We use the periodic storage policy (R, S_i) for each retailer $i=1,2$.

The inventory control period, *R*, is composed by *T* time intervals separated by two successive client demand for each retailer $1 \neq 2$.

According to this policy, at the end of each revision period *R* (assumed to be 28 days, according to Meissner and Rusyaeva (2016), if the retailer's stock position (noted PS_i=available stock - demand) falls below a given value, called replenishment level, then a replenishment order is initiated from the central repository so as to bring that stock position back to initial position. The order is received at the end of the supply period.

The quantity of supply within time intervals *T* is then expressed by the equation (1).

$$Q_i = \begin{cases} S_i - PS_{iT} \text{ if } PS_{iT} \leq S_i \\ 0 \text{ Else} \end{cases} \quad \textbf{(1)}$$

The demand D_i at the retailer *i* during a period *R* is a random variable that follows the normal distribution with mean μ_n and standard deviation σ_n. We make the assumption that the demands at the retailers are independent and identically distributed (*i.i.d*).

When this demand causes a stock out during the period of the check at the retailer *1*, then a transshipment will be made from the retailer *2* to *1*, the amount of transshipment will be noted by X_{21}.We suppose, too, that the transshipment time is zero and that the unit cost of the transshipment noted *C* is a linear cost according to the amount transferred between the retailers. Finally, we assume that partial satisfaction of customer demand by the retailer is not allowed and that claims that can't be satisfied by the available stock and the transshipment are lost and are subject to a cost of break noted $C_p$per unit lost. In all cases, the available stock becomes zero and will remain zero until the next supply.The mathematical modeling that we study in the following paragraph, concerns a transshipment system composed of two non-identical retailers. The approach we have adopted is inspired from the work of (Emel and Lena, (2017)). Recall that these researchers solved a problem of a stock system (R, S_i) by considering a single central repository and two retailers.Our goal is to begin by identifying the difficulties to be met by the resolution of an inventory system by introducing transshipment in order to identify procedures of resolution for a large number of retailers.

1.1. Modelingandexperimentation

The resolution of our problem is fundamentally based on the probabilistic behaviour of customer demands. It is a continuous distribution which follows the normal law.

For this, among the sampling techniques that allow an exploration of customer demand, we selected the simulation. Its principle is then to select, for each demand, random values determined according to an average and a standard deviation. In addition, the demand is generated independently time and between retailers.

We will then, model in this paper, Two-Retail Stock Distribution System, successively appointed "Without-Transshipment" and "With-Transshipment". The latter, may be in the form of a transshipment policy called "Complete-Pooling ", if the retailer agrees to transfer all of its available stock if necessary, or by " Partial-Pooling ", if the transshipment is carried out by preserving a targeted stock level, by modifying the threshold beyond which the retailer agrees to apply transshipment for each experience. First, the latter will be set at a value which is equal to two times the demand (this is that is to say, to protect the next two demand), then it will be equal, to the next demand and finally it is worth to a safety stock which equals 30% of stock position.

The notations used in this paper are as follows:

$\boldsymbol{n}$: *Number of retailers;*

$\boldsymbol{i}$: *Retailer index (counter) with i = 1, 2;*

$\boldsymbol{D_{iT}}$: *Demand during the periodicity T at the retailer i (random variable) follows the normal law* $(\boldsymbol{\mu_i},\boldsymbol{\sigma_i})$*.These demands are independent and identically distributed (i. i. d);*

$\boldsymbol{Q_i}$ *: The quantity of supply for the retailer i;*

$\boldsymbol{R}$: *Inventory position revision period, which is divided into k intervals of time of periodicity T;*

$\boldsymbol{S_i}$: *Maximum level of stock at retailer i at the start of the supply cycle;*

$\boldsymbol{PS_{iT}}$: *Stock position at retailer i at each time period T;*

$\overline{\boldsymbol{TDG_i}}$: *Average Global Desservice Rate for i retailers;*

$\overline{\boldsymbol{\Pi_i^G}}(\boldsymbol{X_G})$: *Average Global Profit for the two retailers i, with i = 1, 2.*

$\boldsymbol{V_i}$ *: Unit selling price for each site i, with i = 1.2.*

$\boldsymbol{C = C_{12} = C_{21}}$*: Unit cost of transshipment whatever the direction of lateral transfer,*

$\boldsymbol{C_{pi}}$*: Unit cost of shortage for such a site i.*

1.1.1. Case "Without-Transshipment"

Conceptual Model

In this case, if the retailer is confronted with a random demand and to satisfy it and does not fall out of stock, he must demand the missing quantity from the central deposit.

This can be represented by figure 1.

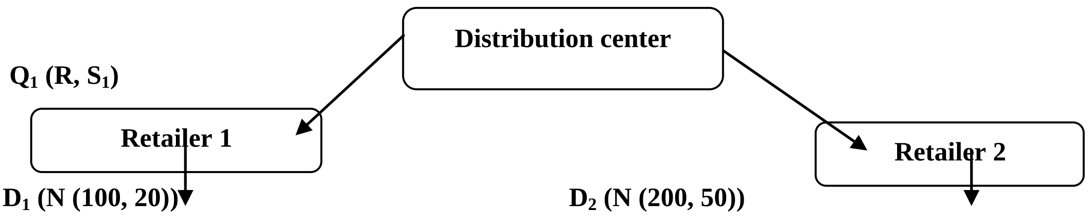

Figure 1.Two-Retailer Stock Distribution System "Without-Transshipment"

For the Without-Transshipment (***No-Pooling***) case, the modeling by the ARENA 16.0 software can be presented by the figure2.

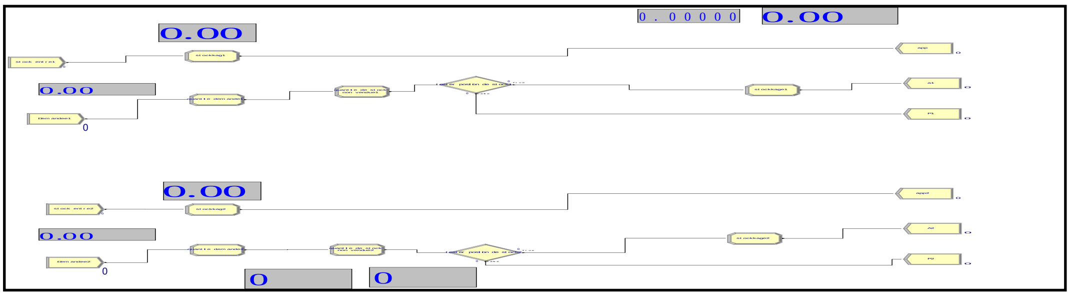

Figure 2.The simulation model Supply Chain: No-Pooling

Assumptions

To properly model this stock system using the Arena software, it is necessary to list the assumptions and the mode of operation retained in this work:

- The storage capacity of the central warehouse is infinite;
- Retailer*i*applies the storage policy (R, S_i);
- Partial satisfaction of an order is not allowed,
- Any unsatisfied order will be lost;
- Only one order (emergency according to the central depot) is allowed per supply cycle (at the end of period R); with $R = kT$
- There is no definite order of priority. All customer orders are managed according to the same FCFS (First Coming First Served) priority rule;
- The distribution center has sufficient storage capacity, so as not to introduce availability constraints (Unlimited storage policy);
- At the start of each supply cycle, a size order Q_i, (with $Q_i = S_i - PS_{iT}$) is placed to reach the stock level noted S_i.

Mathematical Function of Average Global Profit

The Average Global Profit function of our centralized inventory system for two "Without-Transshipment" retailers contains the selling price of the customer product and the cost of the shortage.

It takes the general form of the equation 2.

$$\Pi(X_G) = E\left(\sum_{i=1}^{2} V_i (X_i) - C_{pi} \sum_{i=1}^{2} (I_i^-)\right) \quad (2)$$

1.1.2. Case "With-Transshipment"

Conceptual Model

If one of the two retailers is in the out-of-stock position, then cooperation can be established between them to meet their random demand.This collaboration usually takes the form of "Transshipment-Lateral", also quite simply known as "Transshipment" (Figure 3), which allows stocks to be pooled to alleviate the uncertainties relating to demands arriving at sites of the same level.

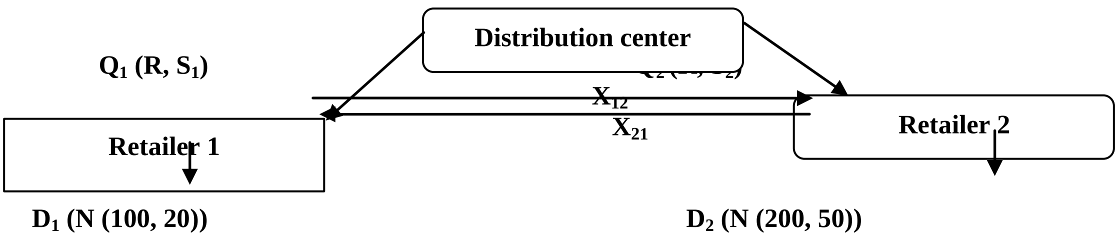

Figure 3. Two-Retail Stock Distribution System "With-Transshipment"

In this section, we will study two transshipment policies successively named "Complete-Pooling" and "Partial-Pooling".

1.1.2.1. Transshipment policies

3.2.1.1.1. "Complete-Pooling"

For the first transshipment policy called "Complete-Pooling" the modeling by the ARENA 16.0 software can be presented in figure 4.

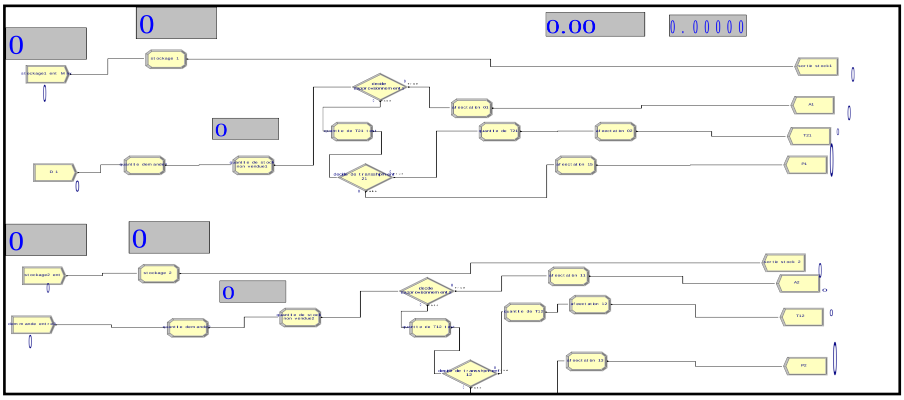

Figure 4.The simulation model Supply Chain: Complete-Pooling

Assumptions

We consider the following assumptions:

- Retailer 1 confronts a random demand independent of demands from retailer 2;
- The transshipment time is zero;
- In the case where a retailer 1 faces a stock shortage, whereas, the retailer 2 has a surplus of inventory, a transshipment of the necessary quantity (X_{21}) will take place from 2 to 1 to avoid or minimize the shortage: this is the correct transshipment (also called reactive transshipment). Otherwise depot 1 may require an emergency order of size Q_1 at the distribution center;
- In the event of "Complete-Pooling", the retailer who is in the overstock position agrees to transfer all of his available stock if necessary.

Mathematical Function of Average Global Profit

The function of Average Global Profit for our centralized system composed of two levels and two retailers, by integrating transshipment and applying the "Complete-Pooling" policy, can be formulated by the equation 3.

$$\overline{\Pi}(X_G) = E\,(V_1(X_1+X_{21}) + V_2(X_2+X_{12}) - C\,(X_{12}+X_{21}) - C_{pi}\,(\textstyle\sum_{i=1}^{2} I_i^-)) \qquad \textbf{(3)}$$

With $X_G = X_1 + X_2 + X_{12} + X_{21}$

Quantity of transhipment

We assume that retailer 1 is the one facing a stock shortage, so according to this transshipment policy, retailer 2 agrees to transfer all of its available stock if necessary, even if this stock is not enough to fill any the demand of the client who is at the origin of the demand for the transshipment.The quantity of the transshipment, according to this policy, will be formulated in the form of the equation (4).

$$X_{21} = \begin{cases} D_{1T} - PS_{1T} \text{ if } D_{1T} - PS_{1T} \leq PS_{2T} \\ 0\, Else \end{cases} \qquad \textbf{(4)}$$

Objective function

The objective is to identify the most economically profitable transshipment policy for a centralized system over a finite time horizon R, by seeking the lowest possible Average Global Desservice Rate.

For this, the objective function of the "Complete-Pooling" transshipment policy will be defined in the form of the equation (5).

$$\text{Max } (E\,(V1(X_1+X_{21}) + V2(X_2+X_{12}) - C\,(X_{12}+X_{21}) - C_{pi}\,(\textstyle\sum_{i=1}^{2} I_i^-)) \qquad \textbf{(5)}$$

S/C

$X_{12} \leq PS_{1T}$, With T= R/k et k=2, 3, 4,…,10.

$X_{21} \leq PS_{2T}$ With T= R/k et k=2, 3, 4,…,10.

$S_i \geq 1$ Strictly positive integer, $\forall i=1, 2$

With

$S_i = (\mu_i * k + \sigma_i \sqrt{k})$, $\forall i=1, 2$ and k :being the number of periodicities, avec k=2, 3, 4,…,10.

And $X_i \sim N(\mu_i, \sigma_i)$.

3.2.1.1.2. "Partial-Pooling"

For the second transshipment policy called "Partial-Pooling" the modeling by the ARENA 16.0 software can be presented in figure 5.

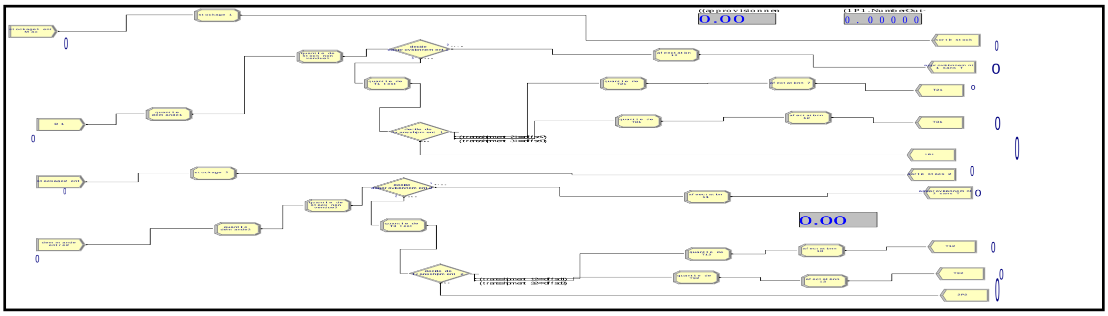

Figure 5.The simulation model Supply Chain: Partial-Pooling

Assumptions

Furthermore, the hypotheses already indicated for the "Complete-Pooling" transshipment policy, we can add another specific assumption for the "Partial-Pooling" policy, that is, the lateral transfer is carried out while preserving a level of targeted stock. We find in research work the following variants:

➢ The retailer accepts the transshipment up to the amount of surplus stock to its safety stock,

➢ the retailer accepts the transshipment up to the amount of surplus stock at his order point,

➢the retailer accepts the transshipment up to the amount of surplus stock at the estimated demand for the following period (See, Archibald et al. (2009)).

➢ the decision to make a transshipment at the level of a retailer depends on the current stock level and the time remaining before the next supply.

In our paper, we are interested in the third variant where the retailer accepts the transshipment up to the amount of surplus demand for a first proposal of the threshold value. Then we add two other personal contributions, first estimating that it will be equal to "Two multiply by demand". Secondly, it will be equal to "30% of stock position", to improve the Average Global Profit of the entire system made up of two retailers while minimizing as far as possible the average stock-out (Average Global Desservice Rate).

In the following sections of this paper, we first describe the mathematical modeling of a "Sans-Transshipment" stock system for a warehouse number set to two. Then we modify it, by integrating, the two policies of transshipment, named, successively, "Complete-Pooling" and "Partial-Pooling".

Mathematical Function of Average Global Profit

The function of the average global profit apply the transshipment policy "Partial-Pooling", requires the integration of the quantity lost for each retailer after the accumulation of stock.

The average global profit function will be formulated by the equation (6).

$$\overline{\Pi}(X_G) = \mathbf{(E\ (V_1(}X_1 + X_{21}) + V_2(X_2 + X_{12})\text{-}C\ (X_{12} + X_{21}) - C_{pi}\ (\sum_{i=1}^{2} I_i^- + X_{P1} + X_{P2}) \quad \mathbf{(6)}$$

With $X_G = X_1 + X_2 + X_{12} + X_{21}$

And X_{P1} : The quantity lost for retailer 1 after the accumulation of stock with partial transshipment.

X_{P2} : The quantity lost for retailer 2 after the accumulation of stock with partial transshipment.

Quantity of transhipment

To significantly improve a purely reactive transshipment policy, it would be possible to combine it with another proactive policy; this will be named by "Hybrid transshipment policy".

In this area of research, the policy of transshipment "Partial-Pooling", to put the action on the importance of the estimate of the future to minimize as soon as possible the quantity not satisfied which governs positively on the economic profitability.

We estimate that retailer 1 is facing an actual stock shortage, therefore, the amount of lateral transfer from retailer 2 to 1 to minimize or avoid this lost quantity, according to this transshipment policy will be carried out while preserving a targeted stock level. named the transshipment threshold and it will be formulated by three equations according to the fixing of the latter.

First of all, we estimate that it will be worth to Twice multiply by the Demand, for this, the quantity of transshipment from 2 to 1 will be formulated by the equation (7).*

If $(PS_{2T} - 2 * D_{2T})$>0 If $(D_{1T}$ -$PS_{1T})$ ≤$(PS_{2T} - 2 * D_{2T})$ So X_{21}=D_{1T} -PS_{1T} **(7)**

Else X_{P1}=$(PS_{2T} - 2 * D_{2T})$ Lost order

Else order lost

Then we assume that this threshold is equal to the Next Demand, and then the amount of lateral transfer from 2 to 1 will be formulated by the equation (8).

If $(PS_{2T} - D_{2T})$>0 If $(D_{1T}$ -$PS_{1T})$ ≤$(PS_{2T} - D_{2T})$ So X_{21}=D_{1T} -PS_{1T} **(8)**

Else X_{P1}=$(PS_{2T} - D_{2T})$ Lost order

Else order lost

Finally, we propose that it be equal to a safety stock which is worth 30% of $\boldsymbol{PS_{iT}}$, therefore, the amount of transshipment will be formulated by the equation (9).

If $(PS_{2T} - 30\% * PS_{2T})$>0 If $(D_{1T}$ -$PS_{1T})$ ≤$(PS_{2T} - 30\% * PS_{2T})$ So X_{21}=D_{1T} -PS_{1T} **(9)**

Else X_{P1}=$(PS_{2T} - 30\% * PS_{2T})$ Lost order

Else order lost

Objective function

For the second transshipment policy ("Partial-Pooling"), the objective function will be defined in the form of the equation (10).

Max (E (V$_1$(X_1+X_{21}) + V$_2$(X_2+X_{12})-C (X_{12} +X_{21})– C_{pi} $(\sum_{i=1}^{2} I_i^- + X_{P1} + X_{P2})$) **(10)**

S/C

$(PS_{2T} - Threshold_{2T})$>0

$(PS_{1T} - Threshold_{1T})$>0

With$Threshold_{iT}$= Twice the Demand, Next Demand and 30% ofPS_{iT}

And X_{P1} : The quantity lost for retailer *1* after the accumulation of stock with partial transshipment.

And X_{P2} : The quantity lost for retailer *2* after the accumulation of stock with partial transshipment.

With T= R/k et k=2, 3, 4,…,10.

$S_i \geq 1$ Strictly positive integer, ∀*i=1, 2*

With

S_i = (?$_i$*k+$\sigma_i\sqrt{k}$) , ∀*i=1, 2*and k :being the number of periodicities, With k=2, 3, 4,…,10.

And X_i~N (?$_i$, σ_i).

1.2. Discussion

We recall that, according to (Meissner and Rusyaeva, (2016)), the initial level of replenishment for a demand that follows the normal law of mean and standard deviation, will take the form of the equation and will be calculated by applying the equation 11.

S_n^0= (?$_n$*T+$\sigma_n\sqrt{T}$) **(11)**

With:
T: number of periods
μ_i: average demand during the period *T* of retailer *i*, with *i=1, 2.*
σ_i: standard deviation of demand of retailer *i*,with *i=1, 2.*

Table 1 shows the different measures of initial stock level of replication and for n= 2, with N: number of retailers.
Recall that the network structure considered in this paper is made up of a distribution center and two retailers, who face random and non-identical demands on average and standard deviation. We assume that the simulation length is 10 years.
We have assumed that the demandD_1 of the first retailer follows the law N (100, 20) and that of the second retailer, D_2 follows the law N (200,50). These demands are Independent and identically distributed *(i.i.d*).
Also, we have considered in all the examples of our research that:

- ✓ The revision period R = 28 days, (Based on (Emel and Lena, 2017));
- ✓ The unit sale price for retailer 1 equal to 95 $ and that of retailer 2 is worth 125 $,
- ✓ The unit cost of rupture whatever the site is equal to 30 $,
- ✓ The unit cost of transshipment =3 $, 0.5 $, k = 2, 3, 4,…, 10.

We led to the resolution of our problem via simulation by successively testing the "Without transshipment" and "With-transshipment" policies. We then give the following performance measures, for the evaluation of the contribution to perform the Pooling between the retailers:
- The number of supply orders (without transshipment),
- The number of orders received with the transshipment application,
- The amount of lateral transfer from a warehouse which is in overstock position to that of the same level which is in rupture position,
- The quantity of order not fulfilled at a retailer (quantity lost),
- Average Global Profit at a retailer,
- The Average Desservice Rate (the rate of customer dissatisfaction after the transshipment).

In table 1 we presentthe different measures of the initial stock level of the replenishment.

Table 1.*Determination of different measures of the initial level of replenishment*

K	$\mathbf{S_1^0}$	$\mathbf{S_2^0}$
2	229	470
3	335	687
4	440	900
5	545	1112
6	648	1322
7	753	1532
8	857	1741
9	960	1950
10	1063	2158

1.2.1. Impact of input parameters on Average Global Profit

We examine the effect of three input parameters on the benefits of transshipment, namely:
- The periodicity "T",
- The unit cost of transshipment,
- And, the threshold of the "Partial-Pooling" transshipment policy.

1.2.1.1. Impact of The periodicity "T"

1.2.1.1.1. "Without-Transshipment" system vs. "With-Transshipment" system

The numbers calculated in Table 1 reveal the considerable effect of collaboration between the sites in terms of Average Global Profit.

Likewise, they present the results of the performance evaluation of the "Complete-Pooling" and "Partial-Pooling" transshipment policies compared to the "Sans-Transshipment" policy. We note, first, that these results verify those already obtained by the mathematical model for a stock system with two non-identical retailers, namely that:
- the comparative values obtained by simulation in Table 1, using the "ARENA" software, confirm the evidence of the advantage of the application of "transshipment" between the sites in terms of improving the Average Global Profit. For example, for k = 2, "Complete-Pooling" improved the performance of the centralized inventory system by increasing the average value of Average Profit Global of the two retailers, from 39125 to 44087, that is to say, a relative change worth 13%,
- These values show the effect of the change in periodicity on economic profitability, by improving the Average Global Profit from k = 2 to k = 4. Whereas, the evolution of the value of the latter undergoes an imperfection beyond k = 4, and this will be explicit for k = 5 up to k = 10,
because, in these periodicities, this profit becomes under the shape of a decreasing curve because of the increase in the number of customer orders by the period R = 28 days.

1.2.1.1.2. "Complete-Pooling" vs. "Partial-Pooling"

Comparative Average Global Profit Improvement Percentage Values Obtained by Simulation Using ARENA Software for the Two Transshipment Policies "Complete-Pooling" and "Partial-Pooling" are reported in Table 2.

Table 2.*Determination of the values of the relative improvement percentage of the Average Global Profit for a unit cost of transshipment = 3 $*

K	Without-transshipment / Complete-Pooling	Complete-Pooling/Partial-Pooling : *Twice the Demand*	Complete-Pooling/ Partial-Pooling : *Next Demand*	Complete-Pooling / Partial-Pooling : *Security Stock=30% of PS_{iT}*
2	13%	3%	17%	26%
3	15%	2%	14%	18%
4	16%	1%	8%	11%

To calculate the different percentages of relative improvement in Average Global Profit indicated in Table 2, we apply the mathematical formula 12.

[% of Relative Improvement= $\overline{\Pi_i^G}$(Complete-Pooling)−$\overline{\Pi_i^G}$(Without-transshipment)) / $\overline{\Pi_i^G}$(Without-transshipment))*100] (12)

(This for the first column of the table while for the two transshipment policies ("Complete-Pooling" and "Partial-Pooling") we apply this formula while looking for the percentage improvement value between them).
From Table 2, we see that, the percentage improvement in Average Global Profit, depends on the periodicity T, as well as, on the transshipment policy applied, ("Complete-Pooling" or "Partial-Pooling").
We note that the first lateral transfer policy ("Complete-Pooling") improves the economic profitability of the "No-transshipment" policy but with a percentage of improvement less than that of the transshipment policy ("Partial- Pooling "). Because of this, the latter is more advantageous, because the Average Global Profit of the former will be improved regardless of the threshold applied. We will conclude, too, that the modification of the latter acts on this improvement, take as an example, for k = 2, the "Partial-Pooling" with a threshold of the "Twice the Demand" target to reach a percentage of improvement Relative Average Global Profit of "Complete-Pooling" equal to 3% but with the change of the threshold to "Next Demand" this value is worth 17% and finally for a threshold equal to "SS = 30% of PS_{iT}" and becomes equal to 26%. By analyzing the variation of the threshold of the "Partial-Pooling" transshipment policy, we note that if the latter is higher than this leads to reducing the chances of supply which results in an increase in the demand no satisfaction rate.
This allows us to conclude that the most economically profitable transshipment policy is that of "Partial-Pooling" and especially with a lateral transfer threshold equal to "SS = 30% of PS_{iT}". Thisobservation leads to a first conclusion in our research, namely that the change of the threshold influences the percentage improvement relative to the Average Global Profit.

1.2.1.2. Impact of the unit cost of transshipment and the threshold for transshipment

The study of the impact of the variation in the unit cost of transshipment on the Average Global Profit is carried out in cases where C = 0.5 $. The simulation results are presented in Table 3.

1.2.1.2.1. . "Without-Transshipment" system vs. "With-Transshipment" system

We examine, for a stock system composed of two levels and two non-identical retailers, the impact of the variation in the unit cost of transshipment and the threshold of the "Partial-Pooling" policy.

Table 3.*Determination of the Average Global Profit for a unit cost of transshipment = 0.5 $*

K	Without-Transshipment	Complete-Pooling	Partial-Pooling		
			Twice the Demand	Next Demand	Security Stock=30% of PS_{iT}
2	39125	45054	46938	52502	56400
3	65044	75754	77740	85952	88976
4	88000	102306	103510	110200	113800
5	86240	101200	102520	105035	107097
6	83600	97657	99125	101257	103356
7	80960	94230	95127	96102	98235
8	78320	91560	93276	94605	97203
9	70400	82359	83900	88007	90102
10	66000	77135	79230	84009	87990

The results of the simulation presented in Table 3 show that a variation in this unit cost of transshipment, by reducing it from 3$ to 0.5$, acts mainly on improving the profitability of the entire centralized system between the "Without -Transshipment "and that of" With-Transshipment ", for that, we will conclude that, the coordination between the sites of the same level allows to improve the profitability of the whole system, but it reaches the most effective values by the application of the "Partial-Pooling" transshipment policy and above all with the fixing of the threshold at "security stock = 30% of PS_{iT}".
For this, we will first of all look for the relative improvement percentage of the Average Global Profit of the centralized system for the first "Complete-Pooling" transshipment policy by reducing this cost (see table 4), then by calculating it with the integration of the second "Partial-Pooling" policy (see table 5).

1.2.1.2.2. "Complete-Pooling" vs. "Partial-Pooling"

The determination of the various relative improvement percentage values of the Average Global Profit for the "Complete-Pooling" transshipment policy or (simply noted CP) between C = 3 $ and C = 0.5 $, is done by applying the equation 13.

$$([\overleftarrow{\Pi_i^G}_{(CP\ for\ C=0.5\$)}]-[\overleftarrow{\Pi_i^G}_{(CP\ for\ C=3\$)}]/[\overleftarrow{\Pi_i^G}_{(CP\ for\ C=3\$)}]*100) \quad (13)$$

Table 4.*Determination of the percentage improvement in Average Global Profit for "Complete Pooling" between C = 3 $ and C = 0.5$*

K	Percentage improvement in Average Global Profit
2	2%
3	2%
4	1%

According to the table 4, we quote for example that, for k = 3 and with a unit cost of transshipment equal to 3$, the "Complete-Pooling" transshipment policy improved the value of average overall profit "Without-Transshipment" from 65044 to 74538, therefore with an improvement value equal to 15%.

But, with a slight reduction in the unit cost of transshipment, this percentage becomes equal to 17%. For this, we will conclude that the unit cost of transshipment has an influence on the improvement of the Average Global Profit of the whole centralized stock system.

Table 5.*Determination of the percentage improvement in Average Global Profit for Partial Pooling betweenC = 3$ and C = 0.5$*

K	Pourcentage du Profit Global Moyen		
	Twice the Demand	**Next Demand**	**Security Stock=30% of PS_{iT}**
2	3%	2%	2%
3	1%	1%	1%
4	1%	3%	2%

From Table 5, we will conclude that the application of the unit cost of transshipment equal to 1 $ is more profitable in terms of gain compared to that which is worth 2 $, and this is remarkable from the results presented in this table, but with a small percentage of improvement.
For example,
- For a threshold = Twice the Demand: the "Partial-Pooling" transshipment policy with a unit cost equal to 0.5 $ makes it possible to improve the Average Global Profit of the one that equals 3 $ with a minimum value equal to 1% up to a maximum value equal to 3%.
- For a threshold = Next Demand: With a unit cost of transshipment worth $ 0.5, the "Partial-Pooling" transshipment policy has improved the Average Global Profit by that which equals 3 $ with a minimum value equal to 1% up to a maximum value equal to 3%.

- For a threshold = 30% of PS_{iT}: With a unit cost of transshipment equal to 0.5$, the economic profitability of the centralized system for the policy of transshipment "Partial-Pooling" undergoes an evolution compared to that which equals 3 $ of a minimum value equal to 1% up to a maximum value equal to 2%.

In fact, according to the study of the impact of the change in the unit cost of transshipment and the threshold of the "Partial-Pooling" policy on the improvement of Average Global Profit, the analysis of the sensitivity of performance to this variation can be summarized as follows:
- The decrease in the unit cost of transshipment influences the increase in the percentage of relative improvement in economic profitability.
- The evolution of Average Global Profit has a strong relationship with the modification of the threshold beyond which the retailer accepts the transshipment to design of available stock.

1.2.2. Impact of the input parameters on the Average Global Deservice

Impact of the input parameters on the Average Global Deservice Rate (The "T" periodicity and the transshipment threshold). We focus here on determining, the policy of transshipment in a centralized stock system which seeks to improve the Average Global Profit at the two retailers by minimizing the Average Global Desservice Rate as much as possible.

Table 6.*Determination of Average Global Desservice Rate*

K	Without-transshipment	Complete-Pooling	Partial-Pooling		
			Twice the Demand	**Next Demand**	**Security Stock=30% of PS_{iT}**
2	0.500	0.360	0.340	0.120	0.060
3	0.600	0.460	0.432	0.097	0.050
4	0.450	0.159	0.148	0.053	0.032
5	0.670	0.357	0.351	0.157	0.067
6	0.750	0.465	0.457	0.195	0.090
7	0.865	0.525	0.512	0.293	0.120

8	0.925	0.620	0.602	0.387	0.202
9	1.005	0.770	0.720	0.492	0.297
10	1.121	0.800	0.795	0.537	0.325

In this section of paper, we formulate the Average GlobalDesservice Rate for the two retailers by equation (14).

$\overline{\mathbf{TDG}}_{\mathbf{i}}=E(\sum_{i=1}^{2}(\mathbf{I}_{\mathbf{i}}^{-} / \mathbf{D}_{\mathbf{i}}))$,(this is the Average Global Desservice Rate). **(14)**
From the analysis in table 6, we note that the Average Global Desservice Rate has a strong relationship with the change in periodicity and it increases beyond k = 4, and becomes in the form of a increasing curve.
But, we analyze the effect of transshipment policies on the minimization of the Average Global Desservice Rate. We notice then that, the first transshipment policy "Complete-Pooling" aims to decrease the rate of the quantity of customer orders not satisfied whatever the periodicity and for example for k = 2 this reduction is worth from 0.500 to 0.360 and that the second “Partial-Pooling” policy aims to reduce it as soon as possible and this will be explicit for the last transshipment threshold which equals ***Security Stock=30% of*** $\boldsymbol{PS_{iT}}$.
We then note that the collaboration between two sites 1 and 2 increases the probability of cycles without shortages in each warehouse by the quantity of transshipment transferred planned from 2 to 1 and likewise from 1 to 2 for an increase in the quantity to order for the site 1 and for site 2. Which results in the probability of customer satisfaction improves after the application of the transshipment.
We will then conclude that, the level of service in a collaborative network is higher compared to the network of independent sites and this plays a very important role in decreasing the amount of lost order. This implies that the economic performance of the group of employees does not only depend on the characteristics of each isolated site, it also depends on the characteristics of each retailer and its relationship with the other depots that make up the inventory system and especially when the cost of transfer lateral is weak. This conclusion should be taken into account in the training of employee groups.

2. Example of Lateral Transshipment in SOTUPLAST Supply Chain

Lateral transshipment is a crucial concept in supply chain management, helping to optimize logistics and reduce costs. In this example, we will explore how SOTUPLAST, a company specialized in the production of plastic materials, implements this strategy to improve its operational efficiency.
Lateral transshipment involves moving goods from one means of transport to another without going through traditional storage. This helps to minimize transit times and optimize the use of available resources.
SOTUPLAST, a major player in the plastic sector, is facing a growing demand for its products, especially plastic packaging and parts. To meet this demand while maintaining high profitability, the company has adopted lateral transshipment as an integral part of its supply chain.
The application of Transshipment-Lateral in SOTUPLAST, results in the following advantages:
- Logistics Infrastructure: SOTUPLAST has invested in strategically located logistics platforms, facilitating direct transfer between trucks and sea containers. This reduces unnecessary travel and waiting times.
- Tracking Technology: The company uses advanced software to track shipments in real time, thus enabling efficient coordination between different modes of transport.
- Strategic Partnerships: By collaborating with regional carriers, SOTUPLAST can quickly and efficiently transship its products to different markets without the need for intermediate storage.

The application of lateral transhipment at SOTUPLAST has led to several positive results:

- Cost Reduction: By reducing the need for storage space, the company was able to significantly reduce its logistics costs.
- Improved Delivery Times: Delivery times were significantly reduced, thus increasing customer satisfaction.
- Increased Flexibility: The ability to quickly adapt transport routes according to demand has allowed SOTUPLAST to remain competitive in the market.

Lateral transhipment has proven to be a winning strategy for SOTUPLAST, allowing it not only to improve its operational efficiency, but also to better serve its customers. By investing in adequate infrastructure and adopting modern technologies, the company has been able to take advantage of this method to optimize its supply chain. This case perfectly illustrates how a well-thought-out logistics strategy can make a difference in a competitive environment.

Conclusion
Flow management within a distribution network is fragile because of the random nature of customer requests. The analysis with a view to increasing its performance and improving its robustness leads to studying models

based on probabilistic representations linked to customer requests. In addition to the approaches developed for stock management locally at each site, companies are moving towards the development of complementary approaches to flow management, within the network, based on the principles of cooperation and collaboration.
The harmonization of storage sites leads to making important decisions (choice of the policy and parameters of applied stock management, cooperation policy and parameters, etc.) in order to guarantee a minimum Average Global Unservice Rate.
The work developed in this thesis is situated in this context. We sought to explore the possible advantages of implementing corrective transshipment as a mode of cooperation between storage sites of the same level. Our objective is to improve the performance of the entire system and not to make a local decision at each storage site, based on an in-depth analysis of solutions related to the improvement of inventory systems with transshipment, for various distribution network structures.
Due to the complexity inherent in the nature of the transshipment problem, we have progressively conducted our study for two different network structures:

✓ Two-tier inventory system composed of two non-identical retailers in terms of mean and standard deviation,

✓ Two-tier inventory system and multiple retailers (n≥3), identical in terms of average demand per period and standard deviation.

✓ For each of these structures, we aimed to determine, over a finite horizon of periods, the transshipment policy ("Complete-Pooling" or "Partial-Pooling") that improves the average overall profit of the system while guaranteeing a desired Desservice rate.

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