RESEARCH ARTICLE

PORTFOLIO SELECTION AND OPTIMAL DECISION REGARDING MARKOWITZ AND SHARPE MODELS

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Abstract : This study aims to build an optimal portfolio and make a comparison between Sharpe's Single Index Model and Markowitz's Mean-Variance Optimization model in selecting the optimal portfolio for the Iraqi Stock Exchange as an initial stock exchange market. The comparative analysis method was used.The study population represented all companies listed in the regular market onthe Iraqi Stock Exchange, which are 93 companies during the period 2012 to 2019.The study sample includes49 companies distributed among the market sectors.The monthly return of the stocks was used as the study analysis. Uses multiple research methods to support hypotheses optimization techniques and statistical analysis. Several results were reached. The main one is thatthe Sharpe portfolio outperformed the Markowitz portfolio in return but atahigher risk.In addition, the Sharpe portfolio outperformed the Markowitz portfoliowhen calculating the Sharpe ratio.Statistically,a difference was found between the return of the two portfoliosin favor of Sharpe'sportfolio. The study recommends investors in the Iraqi Stock Exchange adopt mathematical models, especially the Sharpe model when making their own decisions aboutselecting and building investment portfolios.

Keyword: Portfolio Selection, Sharpe's Single Index Model, Markowitz's Mean-Variance Optimization Model, Sharpe Portfolio, Investment Decision, Iraqi Stock Exchange.

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INTRODUCTION

The dynamism of the investment environment has driven investors to use all the tools and strategies at their disposal to improve their investment decisions. Financial crises in various economic sectors have also contributed to investors becoming more aware of the need to diversify their assets to prevent risks that may be detrimental to their investment (Fawzi, 2014).As a result, the investing decision to select a portfolio is one of the most crucial decisions that investors make because of the implications for their financial situation. Thus, investors resort to a comparison of the returns and risks associated with this decision and are guided by financial indicators and expert opinions about the direction of their decisions, especially since the majority of these decisions are made under conditions of uncertainty or based on possibilities.

The range of investment possibilities available to investors, along with the unpredictability of their investment selections, necessitates a concerted effort on their part to make the best decision in the building of their investment portfolios.

The trend towards private investments in Iraq began after 2003, so the Iraq Stock Exchange (ISX) was established in 2004, which is one of the newly established markets, as it opened the way for investors to trade shares of companies, buying and selling, and stimulating investment in all sectors by directing the community's financial surplus to investments in stocks, bonds, and other financial instruments benefits them, society, and the economy of the Iraqi state in general (http://www.isx-iq.net).

This market is currently experiencing a noteworthy increase in the number of firms listed in it, in addition to the condition of instability in Iraq, where the Iraqi market has been greatly affected by the economic stagnation and the financial crisis that the Iraqi economy has been experiencing since 2014 due to the continuous decline in global oil prices, the cost of the war on terrorism, and other economic accumulations that led to a drop in the growth rate in the gross domestic product to (0.5%), high unemployment rates to 50%, poverty to 25%, inflation to 2%, and a decrease in the savings rate of the Central Bank of Iraq to 20%. The proportion of hoarding of the total cash mass of citizens increased, and this was reflected in the business market. All of this led to a decline in stock prices for the banking sector and other sectors in the trading market, at rates ranging between (50% - 85%) (Union of Arab Banks, 2020;Nima & Obaid, 2018), which constituted a burden on investors in the market.

On the one hand, more businesses want to be compared, which represents this burden; on the other, investors' concerns about the future of their investments are expanding. More research and analysis are needed to allay investors' concerns and give them the data and models they need to make informed judgments.Because the Markowitz and Sharpe models are well-known for analyzing the performance of investment portfolios, the ultimate goal of portfolio creation is to develop a portfolio that gives maximum returns with the least risks by leveraging the benefit of diversification. As a result, according to Markowitz, the correct and accurate selection of the securities included in the portfolio should be based on diversification, and the correlation coefficients between securities should be as few as possible, as this technique reduces risks more effectively than the simple diversification policy (Hindi, 2015).

Because Markowitz's model (MVO) is based on historical data, it requires lengthy calculations when comparing a large number of financial assets over multiple periods (e.g., calculate covariance). Sharpe's Single Index Model (SIM) decreases the number of estimations needed, and the Sharpe Ratio gives a better foundation for gauging performance and is less reliant on historical data.

The two models have been compared, but it has not been determined which one is the most effective for assessing the performance of investment portfolios.Whereas (Putra & Dana, 2020; Akhter et al., 2020; Mandal, 2013; Al-Hasnawi, 2012; Al-Ali & Al Halaseh, 2009; Paudel & Koirala, 2006; Al-Qudah et al., 2004) agreed on the Sharpe model, another group (Badiar, 2019; Ofikwu, 2019) agreed on the Markowitz's model, and others admitted (Suresh & Harshitha, 2017; Sen & Fattawat, 2014) no differences between the two models. As a result, the purpose of this research is to pick the optimal portfolio and compare the Markowitz MVO, and Sharp SIM model to evaluate which performs best for portfolio selection based on the Iraq Stock Exchange.

The objective of this study is to apply the average variance and single-index models to select a stock portfolio for all stocks listed on the market. And, by comparing the optimal portfolios selected by the two models, the researchers hope that accurate information will be provided to investors to support their investment decisions, whether to buy, sell, or hold stocks, and to reduce the level of confusion and fear of selecting the optimal portfolio, particularly in conditions of instability that weaken the predictive ability of sectors that may be affected.

This information also contributes to increasing the financial knowledge of investors, especially in the optimal selection of the components of the stock portfolio, in addition to the importance of this study for shareholders, portfolio managers, and all parties related to the investment process. It also aims to answer the following two study questions:

Is Sharpe's single-index model better in building the optimal portfolio in the Iraqi Stock Exchange when compared to the Markowitz mean-variance model according to the return, risk, and the Sharpe ratio?

Is there a statistically significant difference at the level ($\alpha \leq 0.05$) between the use of the SIM and the MVO on the decision to invest in the Iraqi stock market?

Literature Review

The investment portfolio is a relatively new financial and banking tool in the financial industry, investment world, and capital markets. It is significant and advantageous in terms of the nature of diversity in investment tools, as well as lowering investment risks and contributing to the revitalization of financial and economic markets that are characterized by rapid advances in light of information and communication technology (Marchev, 2013).

Interest in portfolio investment began when Markowitz was preparing his doctoral thesis in 1952.Markowitz assumes that the investor wishes to maximize the portfolio's expected return for any given level of risk. Achieving a higher expected return entails taking on more risk in portfolios that match this condition, thus investors must trade off between risk and expected return.The Efficient Frontier is a graph that depicts the connection between expected return and risk for effective portfolios. All efficient portfolios are well-diversified, as indicated by a point on the effective frontier. A highly risk-averse investor will retain a portfolio to the lower left of the effective boundary, whereas a risk-averse investor will keep a portfolio to the upper right of the effective boundary (Markowitz, 1952,1959).

Markowitz diversification is described as having assets with low correlation to minimize risk without losing portfolio returns. Markowitz was the first to provide accurate content on the concept of diversity. Markowitz's diversification is founded on the fundamental premise that the risks of a portfolio are determined not only by the risks of the assets contained within it but also by the relationship between the returns of those investments. In other words, when selecting the investments that comprise the portfolio, it is necessary to consider the nature of the correlation between the returns generated by it, and this indicates that there is a missing element in measuring the portfolio's losses, and what indicates the relationship between the returns generated by it (Guesmi et al., 2019; Liu, 2019).

It is useful to note that Markowitz's portfolio selection theory is normative as the standard theory is "a theory that describes a standard of behavior that investors should follow in building a portfolio in general. The risk of a modern financial portfolio can be measured using different mathematical formulas. Also, the concept of diversification can be adopted to reduce risk by choosing a properly weighted group of investment assets that are put together by several factors including the risk of investment regression in all asset classes or an individual asset class.

Several complaints have been directed at this model, including its reliance on historical data and the necessity to do computations that take a long time when comparing many investment assets over multiple periods, such as calculating the covariance (Patel & Chakraborty, 2017).

So, the Capital Asset Pricing Model (CAPM) was developed independently by three financial scholars, William Sharpe, Jan Mossin, and John Lintner between 1964 and 1966. It was, in a sense, a pricing model that was "waiting" to be developed, as after Markowitz's advances in portfolio theory in 1952 how an individual asset contributes to overall portfolio risk was well understood. What remained was to link this risk contribution to reward, that is, expected return (Munechika, 2003). This is what the CAPM does, albeit with several constraining assumptions.

CAPM depends on the relationship between return and risk.According to the model the risk is intended as market risk, which is measured by a beta coefficient (β).So, it affects all securities in the market because it cannot be avoided by diversifying the financial portfolio.Therefore, the investor makes a comparison between portfolios based on the return and sksir. The portfolio with the largest return is chosen when its main objective is to obtain the largest return (Al-Nuaimi & Al-Tamimi, 2009).

The Single Index Model (SIM) was developed to build the investment portfolio using fewer inputs naht are used in the MV model and the main assumption of the SIM model is that the variance in securities can be explained by a single factor known as the Index (Lala, 2014).

The SIM is an empirical description of stock returns, based on observations. It is useful to model the risks of a bunch of stocks simply. SIM is interpretable as a special case of the CAPM. Where the CAPM is an economic theory that says that Alpha in the long run has an expected value of zero, which means that the returns investors get are solely due to their exposure to the 'market factor'. This is justified by some reasoning like "other risks can be diversified away, so they will not be rewarded in equilibrium, only 'systematic risk' will be rewarded". However, this has not held up well and it seems that other factors are 'rewarded' in practice. So, the CAPM is seen by many as flawed in some ways (Munechika, 2003).

We could say that Single Index is just a synonym for standard regression (a statistical technique) while CAPM has an equilibrium model (economic model) behind it. Also, the "index' used in Single Index could be any index that the investor think has a good R^2, the index in CAPM is supposed to be the Market Portfolio of all risky assets, a very special portfolio.CAPM assumes that the market is perfect, which is not so the case of SIM as it has some

randomness associated with the return on the security. Therefore, this study used SIM instead of CAPM to select the sharp portfolio and compare it with the MV portfolio(https://quant.stackexchange.com).

Sharpe also calculated the so-called Sharpe ratio to measure the risk-adjusted return on investment in 1966. It is a composite measure of the performance of a stock portfolio based on measuring the amount of excess return per unit of risk related to the portfolio during a certain period (Broquet, et al., 2004). That is, it measures the return as a reward for volatility in the stock price (Guo et al., 2018). It is used to evaluate the performance of investment portfolios and determine whether the profit is a result of good investment decisions or because of taking high investment risks (Sharpe, 1966). He explained that portfolios with a higher average return get more volatility. Representing this relationship with a linear relationship, where Treynor's model was used in evaluating returns, but he replaced the Beta risk with the value of the standard deviation, and this gave a different meaning, as Treynor's model takes only systemic risks into account, while Sharpe takes the total risk in evaluating the return. To measure the performance of each portfolio this research used the Sharpe ratio which from an investor's point of view, describes how well the return of an investment portfolio compensatesfor risk-taking (Schmid & Schmidt, 2009)

The researchers argue that understanding the Sharpe scale is not difficult, The higher the ratio, the higher the performance of the portfolio. Whereas the value may be negative if the return without risk is greater than the average annual return, which means that the performance of the portfolio is less than the performance of the stocks without risks, and the performance of these stocks is often low.

Due to limitations of studies in the Arab Region Stock Markets that used SIM. The current study employsMVO and SIM in building the optimal portfolio. By reviewing the previous studies applied to the world (e.g.,Saleh & Sarhan, 2020 (Syria) Badiar, 2019 (Morocco); Bani- Hamad et al., 2018; Alqadi, 2016 (Jordan); Abbou & Bouflih, 2017 (Saudi Arabia); Abo Amshaa, 2017; Shebeer, 2015; (Palestine); Al-Ardhi & Jaafar, 2016; Al-Hasnawy, 2012 (Iraq)). Badiar (2019) employed MVO and SIM and used the solver optimization technique to select the optimal portfolio within four years period and 23 shareholding companies in Casablanca financial market-Morocco.Its results showed the possibility of building an optimal investment portfolio using Solver software in the Casablanca Stock Exchange. Based on maximizing the Sharpe ratio,the Optimum riskyportfolioobtained after using Solver software, the Markowitz portfolio outperforms the Sharp portfolio. Shabeer (2015) studied the (27) companies listed on the Palestine Stock Exchange. The results of this study showed that the investment portfolios based on the Sharpe model are financially better than both the market portfolio and the portfolios built on a model basis (Treynor, Jensen), in terms of average monthly return and the risk-adjusted average return achieved for six months, as it was found that there are no statistically significant differences due to the type of portfolio between the Sharp portfolio and the market portfolio, in terms of return, risks, and risk-adjusted return.The results of Al-Ardhi and Jaafar's (2016) portfolio whichcontains 21 industrial companies agreed with Markowitz (1952) regarding the tradeoff between risk and return. When diversifying this portfolio, a weak correlation was found between the returns of the stocks that compose it.The results of the study of Abbou and Bouflih (2017) showed that the performance of the four portfolios under study when using the Sharpe Index is better than the performance of the market portfolio. The results of Abo Amshaa (2017) showed that the strategy of selecting stocks and holding them for one year leads to the outperformance of the bank and market portfolio performance over the performance of other portfolios during the entire period of the study. The study also proved that investors who choose stocks in the banking sector and market portfolios outperform the rest of the portfolios, which is what did not happen to investors who choose the stocks of other portfolios.The Mean-variance strategy generated the highest Sharpe ratio, followed by equally weighted, minimum variance, and market value-weighted strategies. The marketvalue-weighted strategy has the highest coefficient of variation which indicates that its returns fluctuate more severely than the optimized and equally weighted strategies. On the other hand, the mean-variance strategy has the lowest variations as compared to the other three strategies. The minimum variance strategy resulted in the lowest return but not the lowest risk as measured by standard deviation which ranks it third in terms of coefficient of variation (Saleh & Sarhan, 2020).Damascus Securities Exchange (2018) conducted a study to develop and compare the performance of two portfolios: an optimal financial portfolio and a weighted market values portfolio. According to the study, the Syrian banking sector has the best investment portfolio.In a study of the portfolio of sub-indices in the Amman Stock Exchange, the results showed that some of the infrastructure sub-indices such as health care and educational sub-indices dominate the optimal selection of the portfolio, while others, such as the hotels and transportation sub-indices, are undesirable (Gharaibeh, 2019). Bani-Hamad (2018) aimed to test the ability of Sharpe's SIM to build the optimal portfolio. This study used the daily returns of companies listed on the Amman Stock Exchange for the period 2013-2017. The proposed model was built by calculating a cut-off point. The results of Sharpe's single-

indicator model showed that the optimal portfolio A daily return of 35%, a daily risk of 0.75%, as measured by the variance of returns.Al-Hasnawy (2012) aimed to derive and estimate the single-index model and to show its relations to diversification, as well as to build the optimal risky portfolio. The results of the study showed that the SIM contributes to solving the problem of choosing the optimal risky portfolio in a clear, simple, and direct way, and provides a clear idea of the nature of the solution other than that provided by the Markowitz model. Sharp is the most widely used model in simplifying building the optimal portfolio of the Markowitz model and following the best methods for estimating and modifying its parameters, especially the "beta".Gharaibeh,(2019)aimed to test the ability of Sharpe's SIM to build the optimal portfolio. It used the daily returns of companies listed on the Amman Stock Exchange for the period 2013-2017. The proposed model was built by calculating a cut-off point. The results of Sharpe's SIM showed that the optimal portfolio achieved a daily return of 35%, and a daily risk of 0.75%, as measured by the variance of returns.Akhter et al. (2020) andYahayah and Ikani (2020) used the same methodology for calculating the cut-off point.The results of the first study showed that by using the SIM, it was found that there is a difference between the risk ratios in the portfolios according to the stocks of the companies selected within the portfolio, where a difference was observed between investing in cars and banks, and in general, it was found that the Sharpe SIM is the simplest widely used method for calculating the optimal portfolio, and it needs to fewer variants compared to the MV model.The second study showed that the SIM is an accurate and sufficient method for choosing the optimal portfolio for investors.

Sen and Fattawat (2014) study resulted in a statistically significant difference in total risk between an optimal portfolio based on SIM and the optimal portfolio based on MVO, while there was no statistically significant difference in the return of the two portfolios. The research of Suresh and Harshitha (2017) proves that both models give approximately the same value for individual return and risk, as well as portfolio return and risk. The research concluded that the Sharpe model is best suited for calculating portfolio risk. Putra and Dana's (2020) study agreed with the study ofSuresh and Harshitha (2017), whose results showed that the performance of the optimal portfolio model using the SIM has better performance than the Markowitz model, but statistically, there is no significant difference in the average return using the single-index model and Markowitz model. The results showed that the SIM is an accurate and sufficient method for choosing the optimal portfolio for investors.

Methods

This study followed the quantitative comparative analysis approach. The study population consists of 93 companies listed no the ISX during the study period. It includes eight sectors (Banks, Insurance, Hotels and Tourism, investment, Services, Industry, Communications, and the Agricultural Sector**)**http://www.isx-iq.net.

As for the study sample, it is a purposive sample that includes all sectors and is in line with the requirements of forming investment portfolios according to the study models. The process of selecting the study sample was subject to the conditions:

1. Continuous listing of the company's shares for the length of the study period from the beginning of 2012 until the end of 2019. Each company must be listed before the beginning of 2012 and continue to be listed until the end of 2019 at least. A total of (59) companies fulfilled this condition (AlHalaseh et al., 2019; Liu, 2009).

2. Continuous trading of the company's shares throughout the study period. Therefore, any stock that contained missing data for extended periods was excluded (AlHalaseh et al., 2019; Stoyan & Kwon, 2010, 2011). A total of 49 companies fulfilled this condition The analyzed sample stneserper six different sectors, as shown in Table (1).

Table 1: The Study Sample by Sectors

Sector	Banks	Insurance	Services	Industry	Hotels	Agricultures	Total
Number of Companies	15	2	6	12	9	5	49
Percentage	31%	4%	12%	25%	18%	10%	100%

Note: This table reports the study sample distribution and their percentage over the study population.

The monthly closing price for the shares was used during the study period from January 2012 to December 2019. The monthly closing price for January 2020 was taken to compensate for the missing observation due to the return calculation. A total of 96 monthly observations per share and a total of 4704 monthly observations for the study sample were used to build the study models. The market index returns were also used as a proxy for all stocks and the conclusions needed for each stock were drawn to build the Sharpe model.Concerning the website of the Central

Bank of Iraq (https://www.cbi.iq), the average interest rates on treasury bills were calculated to estimate the risk-free rate of return (Rf), which equaled0.05.

Variables Measurement

The stock return was measured using the following formula (1):

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \; 1$$

Where $R_i$the return of stock i, $P_{i,t-1}$ the closing price of the stock iat time t, $P_{i,t-1}$ the closing price of the stock i at time t-1.

After calculating the return of the stock, the variance for each stock σ_i^2 is estimated according to Copeland and Weston (1992) as follows (2):

$$\sigma_i^2 = \frac{\sum_i^n (R_i - E(R)_i)}{n-1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 2$$

To fulfill the requirements for selecting the optimal Markowitz portfolio, the expected return of the portfolio is calculated.To do so, the optimal weights of the assets that make up the portfolio must be extracted using the matrix multiplication as follows (3):

$$E(R_p) = w^T R = [w_1, \dots, w_i] \begin{bmatrix} E(R_1) \\ . \\ . \\ . \\ E(R_i) \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \; 3$$

W is the weight vector of the stock (1, …, j) in the portfolio, and R is the expected return vector of the stock (1, …,i) in the portfolio.

After calculating the return for the portfolio, we calculate the total risk (variance) for a portfolio that includes more than three assets with the following equation (4):

$$\sigma_p^2 = w^T S(W) \quad .$$

The portfolio standard deviation is as equation (5) as follows

$$\sigma_p = \sqrt{w^T S(W)} = \left[[w_1 \dots w_i] \begin{bmatrix} \sigma_{11} \dots \sigma_{1i} \\ . \\ . \\ . \\ \sigma_{i1} \dots \sigma_{ii} \end{bmatrix} \begin{bmatrix} w_1 \\ . \\ . \\ . \\ w_i \end{bmatrix} \right]^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \; 5$$

where S represents the variance-covariance matrix of the covariance between each of the stock's returns in the portfolio. Variance - covariance between the returns of any two different stocks, calculated according to the formula (6)

$$S = \sigma_{x,}$$

The optimal weights for the assets that make up the portfolio are the weights that maximize the value of the Sharpe Ratio for the portfolio, which is calculated by equation (7) as follows:

$$S_p = \frac{E(R_p) - R_f}{\sigma_p} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \; 7$$

A single index modelis a statistical tool used to apply CAPM's insights in the real world. The single index model assumes that the stocks vary together due to the joint movement in the stock market and there are no extra-market effects (i.e., the effects of a fundamental factor) that take into account the joint movement of the stocks. The expected return, standard deviation, and covariance of a single index model represent the common movement of a security. How to calculate these metrics is illustrated by the equations below.

To calculate Sharpe's earnings per share, the returns of each share are regressed with the return of the market portfolio R_m to extract the necessary indicators such as alpha $\alpha_i$and beta β_i for each share.The expected return of the stock i calculated as equation (8).

$R_i = \alpha_i + \beta_i R_m + e_i$ ……8

The expected return of the portfolio measured as equation (9)

$$R_p = \sum_{i=1}^{N} w_i(\alpha_i + \beta_i R_i) \quad \ldots\ldots 9$$

The variance of the stock i and the covariance between the stock's return of i and j are measured as equations 10 and 11 respectively.

$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$ ……10

$\sigma_{ij} = \beta_i \beta_j \sigma_m^2$ ……11

The variance of the stock's return (total risk) consists of systematic risk and unsystematic risk and can be measured by equations 12, 13, and 14.

$Systematic\ Risk = \beta_i^2 \sigma_m^2$ …… 12

$Unsystematic\ Risk = \sigma_{ei}^2 = \sigma_i^2 - \beta_i^2 \sigma_m^2$ ……13

To calculate the portfolio risk according to the Sharpe index, it is derived from the previous equations as follows:

$\sigma_p^2 = [(\sum w_i \beta_i)^2 \sigma_m^2] + [\sum w_i^2 \sigma_{ei}^2]$…… 14

Where σ_p^2 equals the variance of the portfolio, σ_m^2 the variance of the market portfolio, σ_{ei}^2 the unsystematic risk of the stock i, and w_i the weight of the stock in the portfolio.

Markowitz Study Model

To choose the optimum portfolio for Markowitz's portfolio, Excel Solver was used by maximizing the return on the portfolio, so the following model was built:

Max$\sum_{i=1}^{N} \frac{w_i R_i - R_f}{\sigma_i}$

Subject To

$$\sum_{i=1}^{N} W_i = 1$$

$W_i \geq 0$, i=1,…,N …… 15

After obtaining the optimal portfolio of risky assets, this research considers the individual investor and based on the level of risk aversion estimates the value of what should be invested in risky assets and what will invest in risk-free assets, so the following equation is used:

$$y^* = \frac{E(R_p) - R_f}{A\sigma_p^2}$$

where y* represents the proportion of the portfolio invested in the risky assets, and A represents the measure of the investor's risk aversion level.

Sharp Study Model

To choose the optimal portfolio for Sharpe's portfolio, Excel Solver was used by maximizing the return from the portfolio, and maximizing the Sharpe ratio, according to the following equation:

$Sharpe\ Ratio = S_p = \frac{\sum_{i=1}^{N} w_i R_i - R_f}{\sigma_p}$…… 18

So, the following maximization model is built:

Max $\frac{\sum_{i=1}^{N} w_i R_i - R_f}{\sigma_p}$

Subject To

$$\sum_{i=1}^{N} W_i = 1$$

$W_i \geq 0$, i=1,…,n …… 19

For Sharpe portfolio C^* = a , a= constant (cut-off point)

$\frac{\sum_{i=1}^{N} w_i R_i - R_f}{\sigma_p} \geq C*$ …… 20

Finally, the Sharpe Portfolio Ratio given the weights of the Single Index Model and the MVO Model will be determined in Equations 7 and 18**.**

Results

Risk and Return of the Study Sample

Table (2) presents the statistical analysis of the study sample of stocks as the expected rate of return and variance. It is clear from the table that 18 shares out of 49 received a positive expected, at a rate of 36.73% from the study sample. The highest return achieved by AMEFamounted to 12.38%, and the lowest return (loss) was for BUND's share, which amounted to (2.91%). The risk of the stocks, as measured by variance, ranged between 0.0028-1.9797. In other words, the total risk of stocks ranged between 5.29% - 140.7%.

Table 2: Stock analysis according to MarkowitzModel

Co. Code	Mean	Variance	Co. Code	Mean	Variance	Co. Code	Mean	Variance
BCOI	-0.0067	0.0074	SKTA	-0.0002	0.0287	IRMC	0.0303	0.0511
BBOB	-0.0168	0.0150	SMOF	-0.0021	0.0047	PLSH	-0.0045	0.0038
BIIB	-0.0088	0.0055	SMRI	-0.0005	0.0094	HISH	-0.0066	0.0103
BIME	-0.0229	0.0129	SNUC	-0.0121	0.0155	HBAY	0.0153	0.0130
BIBI	-0.0081	0.0082	SILT	-0.0053	0.0141	HBAG	-0.0045	0.0083
BNOI	0.0069	0.0205	SBPT	0.0786	0.8312	HNTI	-0.0037	0.0065
BSUC	-0.0036	0.0028	IMAP	-0.0068	0.0062	HTVM	-0.0045	0.0257
BBAY	-0.0136	0.0121	IMOS	0.0461	0.0739	HKAR	-0.0110	0.0101
BGUC	-0.0149	0.0079	IITC	0.0089	0.0047	HMAN	-0.0046	0.0085
BMFI	-0.0109	0.0219	IBPM	-0.0023	0.0166	HSAD	-0.0065	0.0085
BKUI	-0.0025	0.0071	IBSD	0.0167	0.0153	AMAP	0.0015	0.0583
BASH	-0.0103	0.0095	IIDP	0.0048	0.0146	AAHP	0.0091	0.0846
BMNS	-0.0024	0.0069	IHLI	-0.0024	0.0114	AMEF	0.1238	1.9797
BUND	-0.0291	0.0130	INCP	0.0025	0.0171	AIPM	0.0061	0.0334
BELF	-0.0135	0.0072	IKLV	0.0034	0.0165	AIRP	0.0123	0.0164
NAME	0.0043	0.0239	IIEW	-0.0085	0.0083			
NGIR	0.0027	0.0070	IMIB	0.0202	0.0436			

Note: This table reports the descriptive statistics of the study sample

Table (3) shows the stock returns for the study sample, Ri, measured according to Equation 8, by conducting a simple regression test for the expected returns for each share with the expected return for the market portfolio. The highest return was 12.6% for the AMEF and the lowest (loss) (0.0301) was gained by the BUND stock. The highest unsystematic risk σ_{ei}^2 was 1.9791, and it belonged to the AMEF, and the lowest unsystematic risk belonged to the BSUC, which amounted to 0.0028. By calculating the average beta coefficient for the sample, which is equaled to 40.47%, it is noted that its value was less than 50%, and this indicated that the sample on average is less sensitive to price fluctuations in the market portfolio (Al-Ali & Al-Halaseh, 2009).

Table 3: Stock analysis according to the Sharpe Model

Co. Code	Ri	Alpha	βi	σ_{ei}	σ^2_{ei}	Ri-Rf/βi
BCOI	-0.0120	-0.0100	0.4380	0.0772	0.0060	-0.0274

BBOB	-0.0195	-0.0190	0.1950	0.1200	0.0144	-0.0998
BIIB	-0.0077	-0.0080	-0.1390	0.0732	0.0054	0.0552
BIME	-0.0231	-0.0230	0.0380	0.1136	0.0129	-0.6076
BIBI	-0.0092	-0.0090	0.0690	0.0904	0.0082	-0.1328
BNOI	0.0059	0.0060	0.0530	0.1431	0.0205	0.1109
BSUC	-0.0040	-0.0040	0.0090	0.0530	0.0028	-0.4468
BBAY	-0.0152	-0.0150	0.0920	0.1093	0.0120	-0.1654
BGUC	-0.0187	-0.0180	0.3180	0.0843	0.0071	-0.0589
BMFI	-0.0133	-0.0130	0.1300	0.1467	0.0215	-0.1023
BKUI	-0.0020	-0.0020	-0.0170	0.0842	0.0071	0.1153
BASH	-0.0086	-0.0090	-0.1520	0.0962	0.0093	0.0569
BMNS	-0.0031	-0.0030	0.0530	0.0832	0.0069	-0.0589
BUND	-0.0301	-0.0300	0.0450	0.1140	0.0130	-0.6690
BELF	-0.0127	-0.0130	-0.1080	0.0845	0.0071	0.1180
NAME	0.0040	0.0040	-0.0090	0.1547	0.0239	-0.4468
NGIR	0.0057	0.0050	-0.2910	0.0798	0.0064	-0.0195
SKTA	0.0012	0.0010	-0.0760	0.1691	0.0286	-0.0155
SMOF	-0.0020	-0.0020	0.0030	0.0686	0.0047	-0.6690
SMRI	-0.0038	-0.0030	0.3210	0.0917	0.0084	-0.0117
SNUC	-0.0120	-0.0120	-0.0010	0.1246	0.0155	11.9977
SILT	-0.0061	-0.0060	0.0510	0.1184	0.0140	-0.1200
SBPT	0.0800	0.0800	-0.0140	0.9116	0.8310	-5.7166
IMAP	-0.0071	-0.0070	0.0500	0.0785	0.0062	-0.1423
IMOS	0.0449	0.0450	0.0250	0.2717	0.0738	1.7977
IITC	0.0089	0.0090	0.0490	0.0682	0.0047	0.1813
IBPM	-0.0007	-0.0010	-0.1320	0.1276	0.0163	0.0052
IBSD	0.0158	0.0160	0.0810	0.1235	0.0153	0.1952
IIDP	0.0050	0.0050	0.0130	0.1209	0.0146	0.3823
IHLI	-0.0032	-0.0030	0.0890	0.1065	0.0113	-0.0360
INCP	0.0008	0.0010	0.0880	0.1304	0.0170	0.0090
IKLV	0.0042	0.0040	-0.0830	0.1282	0.0164	-0.0505
IIEW	-0.0092	-0.0090	0.1050	0.0907	0.0082	-0.0881
IMIB	0.0189	0.0190	0.0490	0.2086	0.0435	0.3854
IRMC	0.0311	0.0310	-0.0450	0.2258	0.0510	-0.6912
PLSH	-0.0050	-0.0050	-0.0030	0.0618	0.0038	1.6643
HISH	-0.0083	-0.0080	0.1080	0.1010	0.0102	-0.0764
HBAY	0.0150	0.0150	-0.0160	0.1141	0.0130	-0.9398
HBAG	-0.0038	-0.0040	-0.0800	0.0908	0.0083	0.0477
HNTI	-0.0040	-0.0040	0.0200	0.0808	0.0065	-0.2023

HTVM	-0.0074	-0.0070	0.1600	0.1581	0.0250	-0.0461
HKAR	-0.0111	-0.0110	0.0240	0.1007	0.0101	-0.4607
HMAN	-0.0076	-0.0070	0.2510	0.0890	0.0079	-0.0302
HSAD	-0.0096	-0.0090	0.2570	0.0891	0.0079	-0.0374
AMAP	-0.0001	0.0000	0.0450	0.2413	0.0582	-0.0023
AAHP	0.0100	0.0100	-0.0210	0.2908	0.0846	-0.4785
AMEF	0.1260	0.1260	-0.0170	1.4068	1.9791	-7.4141
AIPM	0.0049	0.0050	0.0640	0.1823	0.0332	0.0758
AIRP	0.0142	0.0140	-0.1060	0.1275	0.0163	-0.1344

Note: This table reports the statistical analysis after regressed with the market index.

To answer the first study question, which states "Is the SIM better in building the optimal portfolio in the Iraqi Stock Exchange when compared to the MVO according to the return and the Sharpe ratio?"
Table (4) shows that choosing the optimal portfolio according to MVO and SIM using Excel Solver 2016 to maximize the return from Markowitz portfolio selected nine high-risk assets (The first asset relates to service Sector S, and the following five assets are in Industrial Sector I, and the last three are in Agriculture Sector, A) while choosing two high-risk assets for Sharpe's portfolio (relate to Service SectorS and Agriculture SectorArespectively). The monthly return of the Markowitz portfolio was 2.645%, and the monthly return of Sharpe's portfolio was 10.23%. The risk of Markowitz's portfolio reached 7.466%, which is less than the risk of Sharpe's portfolio, which reached 85.46%.
The Sharpe ratio was calculated for the two portfolios.The Sharpe portfolio outperformed, as the Sharpe ratio reached 6.12%. They* value equals 89.7 represents the share invested in the risky assets of the portfolio.Therefore, the value of 10.30 remains that can be invested in other assets such as bonds and/or real estate.
The components and weights of both Markowitz's portfolio and Sharpe's portfolio resulting from the use of Optimization Excell Solver also appeared in Table (4). Where Sharpe's SIM portfolio outperformed Markowitz's MVO portfolio in return and at the highest risk, as well as outperformed in the value of the Sharpe ratio. Thus, the study's first question is answered.

Table 4: Selection of the optimal portfolio

MVO Portfolio					SIM Portfolio				
Co. Code	**W_i**	**σ_i**	**R_i**	**R_p**	**Co. Code**	**W_i**	**σ_i**	**Ri**	**Rp**
SBPT	0.042	1.382	0.0786	0.0033	**SBPT**	0.48	0.912	0.0784	0.037632
IMOS	0.203	0.2545	0.0461	0.0094	**AMEF**	0.52	1.407	0.1241	0.064532
IITC	0.281	0.0695	0.0089	0.0025					
IBSD	0.20	0.1261	0.0167	0.0034					
IMIB	0.084	0.21398	0.0202	0.0017					
IRMC	0.091	0.23430	0.0303	0.0028					
AMAP	0.031	0.25336	0.0015	0.00005					
AAHP	0.043	0.31211	0.0091	0.0004					
AMEF	0.025	2.8387	0.1238	0.0031					
Total	**1.00**					**1.00**			
R_p				**2.648%**					**10.23%**
σ_p				**7.466%**					**85.46%**
S_p				**(31.54%)**					**6.12%**

Note: This table reports the compenants of the MVO and SIM portfolios, return, risk, and the sharp ratio.

To confirm statistically the significance of the superiority of the SIM portfolio over the MVO portfolio, the second hypothesis was formulated. The nonparametric statistical test of the Wilcoxon-Mann-Whitney test was adopted to

compare the average return ranks of the two portfolios. For that, the expected return for the two portfolios was calculated using equation 3.
To ensure that the conditions for testing the independent samples were met, the normal distribution of the two sets of the dependent variable was tested using the Shapiro-Wilk and Kolmogorov-Smirnov test. Table (5) shows the results of the two tests which confirms that the returns of the two models do not follow the normal distribution in terms of Kolmogorov-Smirnovand Shapiro-Wilk,where (P-Value < 0.05). The histogram was also used to judge the similarity or difference in the shape of the distribution that showed the difference in the distribution of the returns of the two portfolios.

Table 5: Normal Distribution Results

			Kolmogorov-Smirnov		Shapiro-Wilk	
	Group	**Df**	**Statistic**	**Sig.**	**Statistic**	**Sig.**
Model	1	95	0.419*	0.000	0.205*	0.000
	2	95	0.149*	0.000	0.819*	0.000

*Note: *, indicates insignificance of the distribution*

Table (6) shows the descriptive statistics and correlation tests. It is noted that the average returns of the SIM portfolio are higher than the returns of the MVO portfolio, and the overall risk of the returns of the SIM portfolio is 84.5%, compared to the total risk of the MVO portfolio of 9.5%. These statistical resultsare identical to the optimization result in Table 4. The correlation coefficient for the returns of the two portfolios was 0.498, which indicates a significant medium-positive correlation. It means that when the return of the SIM portfolio rises by one degree, the return of the MVO portfolio rises by about half a degree.The average rank comparison is the basis on which the Mann-Whitney test operates(King & Eckersley, 2019).
Therefore, Table (6) also shows that the average rank of SIM portfolio return, which amount to 86.63, is significantly higher than the average rank of MVO portfolio return, which amountsto 37,104. These values prove the difference between the returns of the two portfolios. For statistical verification, the Wilcoxon-Mann-Whitney test was conducted.

Table 6: Descriptive statistics and CorrelationTests

Portfolio	**Mean**	**SD**	**Rank AV.**	**Correlation**	**P-value**
SIM	0.10212	0.84546	86.63	0.498*	0.000
MVO	0.02648	0.09504	37.104		

*Note: *, indicatessignificant relation*

Table (7) shows the results of the Wilcoxon-Mann-Whitney test, where the Z-value is (2.223) with an acceptable significance of 0.026.So, we acknowledge the existence of a statistically significant difference between the average ranks of the returns of the two portfolios and accept the second hypothesis of the study.

Table 7: Wilcoxon- Mann- Whitney Results

Man-Whitney	**Wilcoxon W**	**Z**	**N**	**Asymp. Sig**
3670	8230	-2.223*	95	0.026

*Note: *, indicates significant difference*

Discussion

Despite being a theoretically rigorous model, practitioners of the single index model, such as Elton, Gruber, and Padberg (1976) demonstrate that the MVO model is difficult to operationalize**.**In this study, 49 stocks are used, hence 49 estimates of return and variance are required, as well as 1176 estimations of covariances ((n2n)/2) (Sarker, 2013). When more stocks are considered, more estimations are required, which takes time and reduces the covariance accuracy. Furthermore, Microsoft Excel cannot be used regularly unless the same number of stocks are used; if the number of stocks changes, the code must change as well. SIM, on the other hand, requires less information. The only data required are 49expected returns, variance, alpha, and beta. The single index model can be utilized regularly, and it takes less time to buildan optimal portfolio (Akhter et al., 2020). Regardless of the model chosen, the results suggest that owning a well-diversified portfolio is preferable to holding a single asset, as

proposed by Modern Portfolio Theory. This is demonstrated in the table (4), where individual risks are greater than portfolio risks when both the SIM and MVO models are used.The MVO model likewise works best if the investor is risk-averse, as Markowitz predicted (1952). This is illustrated in Table 4, where the SIM portfolio risk exceeds the MVO model portfolio risk. A risk-averse investor should utilize the MVO model, whereas a risk-seeking investor should employ the SIM.

Ofikwu's (2019) SIM portfolio is primarily made of construction and manufacturing firms. Kamal (2012) discovered that many of the equities in the portfolio were bank and financial companies by employing SIM. Using SIM, AbuAmsha (2017) concludes that investors who choose equities in the banking sectors and market portfolio beat the rest of the portfolios. However, in this study, the portfolio is primarily constituted of the service and agricultural sectors when utilizing SIM, and industrial and agricultural companies when using MVO. This may refer to the decline in stock prices for the banking sector affected by the economic stagnation and the financial crisis that the Iraqi economy has been experiencing since 2014 (Union of Arab Banks, 2020https://uabonline.org/ar/).

The findings of this study corresponded with the findings of other studies on the preference of the Sharpe model in portfolio construction, such as the studies of (Putra & Dana, 2020; Akhter et al., 2020; Patel, & Chakraborty, 2017; Sen & Fattawat, 2014; Al-Hasnawi, 2012),

It also differed from the results of Bediar (2019), which demonstrated the possibility of building an optimal investment portfolio in the Casablanca Stock Exchange using Solver software, and that an optimal risky portfolio was obtained based on the results of maximizing the Sharpe ratio after using Solver software, and that Markowitz's portfolio outperforms Sharp's portfolio according to the Sharp Ratio. It also disagreed with Ofikwu (2019), who demonstrated in his analysis of the Nigerian Stock Exchange that the Sharpe portfolio performs better for short-term investment periods, but the MVO portfolio performs better for long-term investment periods.

Conclusion

Risk and return are critical considerations when making any investment decision. This research intends to examine the potential for investors in terms of returns and investment risk while investing in the equities of enterprises listed on the Iraqi stock exchange. On the practical side, the research findings were approved. In terms of return, Sharp's SIM portfolio outperforms Markowitz's MVO portfolio. However, Markowitz's portfolio is less risky than Sharpe's portfolio. When the Sharpe ratio for the two portfolios was calculated, the SIM portfolio outperformed the MVO portfolio. This could be because the Sharp portfolio performs better over short periods, whereas the Markowitz portfolio performs better over long periods (Ofikwu, 2019). The analysis revealed that Sharp's portfolio, which consists of only two assets, has high risks, confirming the need for diversification to reduce these risks. Furthermore, Markowitz's portfolio of nine assets achieved greater diversification and lower risk. The fact that the optimal portfolio contains only two assets indicates that the Iraqi Stock Exchange is experiencing high volatility. Statistically, a significant difference was found between the investment decision using the MVO and the SIM.The Sharpe Model appears to perform well on the Iraqi Stock Exchange. Sharpe Model is suitable for risk-taking investors, whilst Markowitz Model is suitable for risk-averse investors.

Based on the study's findings, the researchers recommend that investors conduct a periodic review of the performance of their investment portfolios due to the current state of economic instability in Iraqand that they use mathematical models, particularly the Sharpe model when making decisions about the selection and formation of investment portfolios. The need for the stock exchange's competent authorities to create a database containing the proceeds of investment portfolios for their importance in the preparation of studies and assisting investors in making rational investment decisions (Al-Badran, 2020).

The two researchers recommend conducting future studies of the financial markets at the regional level and comparing long and short-term periods. Conducting prospective studies to test the SIM and compare its performance over time against MVO. Examine additional models not covered in this study and compare their results to the results of this study.

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